Pressure Drop in Geometrically Ordered Packed Beds of Spheres

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The pressure drop of water was measured when water flowed through a bed of stainless steel ball bearings packed in an *ordered* rhombohedral geometry. Experiments were carried out with eleven different packed beds, encompassing the entire range of the square-base array, in the same 10.85 by 10.85 by 30-in. rectangular test column in a forced circulation loop at modified Reynolds numbers up to 17,000. The test variables included water velocity, bed voidage, spacing between adjacent balls, ball diameter, and bed height. Curves of friction factor vs. Reynolds number are presented. An increase in the relative horizontal spacing between balls was found to have a more important effect than an increase in voidage in decreasing the pressure drop. A general correlation relating the mutual effects of bed voidage and ball spacing on pressure drop that would bring all the data points together, especially in the transition flow region, could not be found. As a result, the system appears to consist of two distinct parts separated at the minimum packing density. A correlation was found only for the first, but from a practical point of view more important, region. Data may be corrected for bed voidage, but only for small variations in ball spacing, by the ratios of $(1-\varepsilon)/\varepsilon^3$ at the two voidages. No entrance and exit effects could be measured beyond the first seven ball layers.

A unique method was found at Brookhaven National Laboratory (*1*) that permits one to obtain perfectly *ordered* packed beds of spheres, even when these are dropped randomly into rigid rectangular columns. The spheres are packed in a rhombohedral array, the spheres in any layer forming corners of squares and those in the next layer being located in the cusps formed at their centers. The bottom layer must be oriented on a base which is either an inverted pyramid or a flat plate with suitably spaced countersunk holes. The packing is then continued within the confines of the rectangular column whose sides parallel the *diagonals* of the basic square. This ensures that all the outside spheres of the bed will be tangent to the container walls. This method provides the basis for a nuclear breeder reactor concept—the ordered bed fast reactor. This utilizes packed beds of 1/4-in. diameter spherical fuel particles arranged in a geometrically ordered array in 12 by 12 by 48-in. steel containers, with sodium coolant flowing directly through the bed. More generally, it applies to the packing of solid spheres in any industrial application in which precisely known, high densities of solids, and large heat and mass transfer surfaces are desirable.

One of the principal difficulties encountered in the design of such ordered packed beds is the uncertainty regarding the pressure drop of fluid flowing through the bed. Martin (*2, 3*) and Lydersen (*4*) obtained data on the pressure drop in tightly packed, ordered arrays of spheres, where the ball spacing is just equal to the ball diameter. While there are methods for correcting the bed pressure drop for increased void volume which have been tested in randomly packed beds, Hendrie's preliminary data (*5*) obtained for more loosely packed *ordered* arrays of identical geometry did not agree with the results of Martin and Lydersen when the same correction for voidage was used. The purpose of the work described in this paper, and presented in greater detail in reference 6, was to extend Hendrie's work and measure the pressure drop of water flowing through packed beds of spheres in a 10.85 by 10.85 by 30-in. test column as functions of the following variables: superficial water velocity, 0 to 3.35 ft./sec.; bed voidage, 0.2595 to 0.3198; spacing between adjacent balls in a bed layer, 1.000 to 1.221 diameters; ball size, 1/8 to 1/2-in. diameter; and bed height, one to fifty ball layers (however, three beds were packed to bed heights of eleven, eighty-two, and one hundred ball layers, respectively).

All the beds studied were packed in a rhombohedral geometry, conforming to the designation Rhombohedral No. 3 in the system defined by Graton and Fraser (*7*). The closest center-to-center spacing of spheres in each layer can be varied between 1.000 and 1.225 sphere diameters (with a spread of 0.0603 in packing fraction, that is, from 0.6802 to 0.7405), while still retaining the basic packing array (Figure 1). In the closest packing all spheres in the bed touch their neighbors on all four sides, while at the widest spacing the spheres in any layer are spread just far enough apart so that all the spheres touch their neighbors directly above and below (Figure 2). The packing density goes through a minimum so that the loosest packing (0.6802) actually occurs at an intermediate sphere spacing of 1.155 diameters in each layer. Channels are formed between the spheres and these are continuous through the entire bed, thereby further reducing the pressure drop of liquid flowing through this bed beyond that of a randomly packed bed of the same voidage.

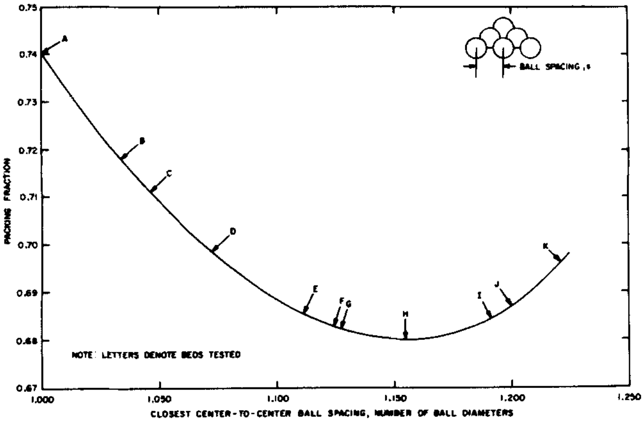

Fig. 1. *Variation of bed packing fraction with ball spacing in a rhombohedral geometry.*

TABLE 1. CHARACTERISTICS OF TEST BEDS

Test bed	s/D_p	ε Bed	ε Wall	ε Overall	D_p, in.
A	1.000	0.2595	0.4775	0.2693	0.25
B	1.034	0.2821	0.4765	0.2915	0.25
C	1.046	0.2890	0.4772	0.3061	0.50
D	1.072	0.3013	0.4711	0.3083	0.25
E	1.111	0.3143	0.4623	0.3212	0.25
F	1.124	0.3171	0.4581	0.3201	0.125
G	1.127	0.3177	0.4583	0.3297	0.50
H	1.154	0.3198	0.4450	0.3246	0.25
I	1.191	0.3155	0.4238	0.3167	0.125
J	1.200	0.3128	0.4174	0.3175	0.25
K	1.221	0.3038	0.4007	0.3120	0.50

EXPERIMENTAL PROGRAM

The experiments were conducted with water flowing downward through eleven different ordered packed beds of balls (Table 1). No fractional balls were placed along the walls, and the voidage here differed from that within the packed bed, although the magnitude of this effect was generally small.

Bed Packing

Highly polished type 316 stainless steel ball bearings in 0.125-, 0.250-, and 0.500-in. diameter sizes were used in the investigation. They were purchased with a factory diametral tolerance of ±0.001 in. and a surface finish of 9 root mean square. Prior to use in this study, the balls were carefully degreased with acetone.

Experimental Equipment

All parts of the test loop (Figure 3) in contact with water, except for the cast iron impeller of the circulating pump, were made of type 304 stainless steel in order to minimize rust formation. The balls were packed in a specially fabricated 30-in. long rectangular test column with flanges at either end to facilitate loop installation. Careful measurements showed that the cross-sectional dimensions of the column were 10.853 by 10.853 in., with an average deviation of ±0.017 in. Stainless steel grid support plates were clamped at the bottom of the column to orient as well as to support the various packings. Countersunk holes in each grid plate served to locate the bottom bed layer; smaller holes were drilled through the intervening areas to permit liquid to flow out of the column. The test column was mounted in the middle of a vertical assembly with 90-in. long capped approach sections at each end, having the same cross-sectional area and shape as the test section. The circulating water flowed in and out of this assembly through loop piping welded at right angles to the approach section, 7 ft. beyond each test column flange. A straightening section consisting of 1/2-in. pipe stubs was installed across the mouth of the approach pipe to reduce the turbulence caused by water flowing through the sharp bends. The pressure taps were located in each of the approach sections, one 12 1/4 in. above and the other 50 1/4 in. below the test column flanges. This location prevented the formation of pressure fluctuations and distortions caused by obstruction of the openings by balls in the bed.

A 100-hp. centrifugal pump circulated the water from a 600-gal. supply tank through the column. The flow was measured with a Hays model 346 electromagnetic flow meter and, at high flows, with an orifice meter. The electromagnetic flow meter was selected as the primary measuring device in these experiments because it could encompass a very large range of flows without necessitating any modifications of the piping circuits. Water flow was regulated with a Powell globe valve.

The pressure taps from the column were connected to two manometers and Bourdon-type Heise pressure gauges mounted in series (Figure 3). U-type Meriam manometers were used, each with a 40-in. scale and 1-mm. graduations. One was inverted and filled with air and the other with mercury, both reading directly in height of water flowing. The Heise gauges, one mounted on each pressure tap, had 16-in. dials with a range of 0 to 150 lb./sq. in. gauge with 0.1 lb./sq. in. graduations, were temperature compensated, and had provisions for venting and pressure damping. They were calibrated at the factory and were accurate to ±0.10% of full scale.

A well-type chromel-alumel thermocouple probe measured the water temperature at the column outlet and was connected to a Honeywell strip chart recorder. Ambient air temperature was measured with a mercury thermometer hung near the manometers.

Procedure for Measuring the Bed Pressure Drop

The centrifugal pump was started with the throttling valve shut off. For a given height of packed bed, the water flow rate was varied from zero to the maximum flow attainable with the

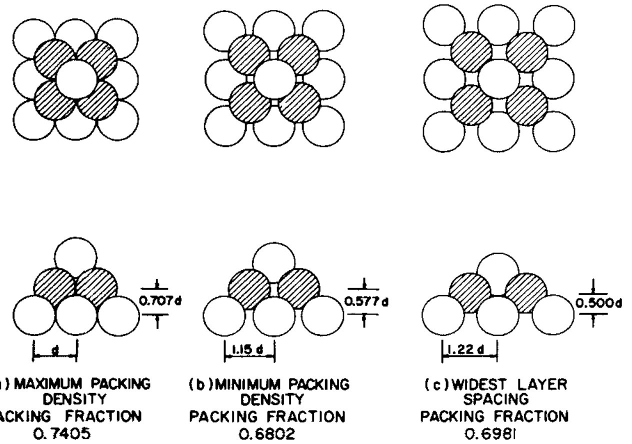

Fig. 2. Limits in the ball layer spacing of the ordered packed bed.

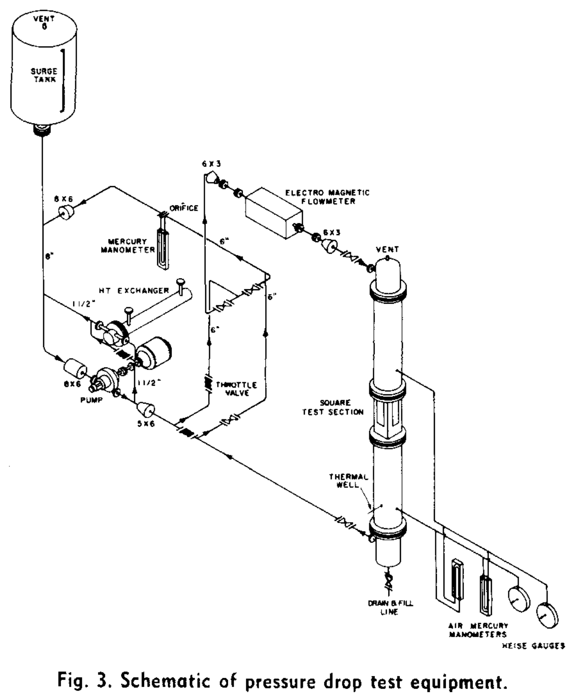

Fig. 3. Schematic of pressure drop test equipment.

pump, and then reduced again, but in fewer increments, to zero by changing the position of the globe valve. Each condition was run in duplicate so that two or four pressure readings were obtained at each flow rate.

The loop, as well as the pressure tap lines and gauges, were carefully purged before each run. The zero readings on the mercury and air manometers were then obtained, the difference in height of liquid in the two manometer legs being usually less than 1 mm. The pressure readings on the air and mercury manometers, and on the mercury manometer and Heise gauges, overlapped. Pressure differences of less than 1.00 lb./sq. in. gauge on the Heise gauges and of less than 0.20 cm. on the mercury manometer were discarded because of damping in the respective instruments. The smallest graduation on the manometers was 1 mm. so that it was possible to read differences in height of as little as 1/3 mm., while that of the Heise gauges was 0.10 lb./sq. in. gauge and it was possible to obtain readings as low as 0.05 lb./sq. in. gauge. The test bed was observed visually through the windows on the column sides to ensure that no air bubbles were visible and that the bed had not shifted. There was no change in pressure drop with time.

Treatment of Experimental Data

The total pressure drop, expressed as head of water in inches, was calculated for the three measuring instruments at each flow rate. The individual meter readings were then averaged at each flow rate and corrected for the difference in temperature between the instruments and the water in the column.

Since the pressure taps were located above and below the test bed, the pressure drop through the grid plate containing the bottom layer of balls would have to be measured separately and then subtracted from the values of total pressure drop obtained for each subsequent bed height. The resulting difference would then give the desired pressure drop through the packed bed only. Unfortunately, the water temperature varied during the run, and such a simple subtraction of the two pressure drops at the same flow rate could not be made. Instead, average values of pressure drop and temperature, obtained from the two or four measurements at each flow rate, were used to calculate G/μ and $\rho(\Delta p_f)/G^2$ at each flow rate and were then plotted. The values of density and viscosity of water were obtained from the *Chemical Engineers' Handbook* (*8*). Using these plotted curves, the values of $\rho(\Delta p_f)/G^2$ for the grid plate and bottom layer of balls could now be subtracted directly from the corresponding values at other bed heights at each value of G/μ. The friction factor was calculated from values of $\rho(\Delta p_f)/G^2$ as

$$f = \frac{g_c \rho D_p (\Delta p_f)}{2G^2(\Delta L)}$$

and averaged for three different bed heights no longer subject to entrance and exit effects. It was then plotted against the modified Reynolds number.

RESULTS AND DISCUSSION

A dimensional analysis of pressure drop through porous beds was reported in the literature (*9*) which leads to the following expression for the friction factor

$$\frac{g_c \rho D_p (\Delta p_f)}{G^2(\Delta L)} = F\left(\frac{D_p G}{\mu}, \varepsilon, \frac{e}{D_p}, \frac{D_p}{b}, \varphi_s, \beta, \frac{D_p}{L}\right)$$

where ε is the bed voidage, e/D_p is the relative particle surface roughness, D_p/b is the wall effect, φ_s is the particle shape factor, β is the bed geometry factor, and D_p/L is the entrance and exit effects.

In the case of the experiments conducted here, the results were based on beds no longer subject to entrance and exit effects, and packed with very smooth spherical particles whose surface roughness-to-ball diameter ratios varied between 2 and 8×10^{-5}. There is essentially no measurable surface roughness effect on pressure drop with these low values and in the range of modified Reynolds number investigated. These variables were therefore not considered further here. The pressure drop for this system is related to the bed voidage and bed geometry in a very complex way because the same voidage may be obtained with two different ball spacings (Figure 1). These two variables are therefore interdependent if the entire range of bed spacings is considered. Since fractional balls were not used in this study, a wall effect was present.

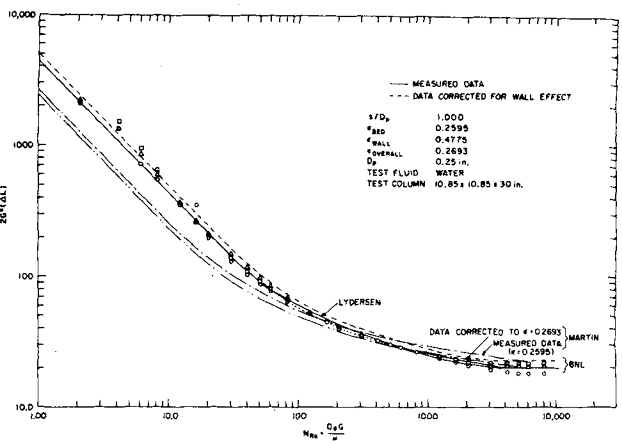

Fig. 4. Variation of friction factor with Reynolds number in the tightest ordered packed bed.

Friction Factor-Reynolds Number Relationship

At the closest ball packing, test bed *A*, good agreement was obtained with the data of Martin (*2*) and Lydersen (*4*) for Reynolds numbers in excess of 100 (Figure 4). Below this, the Brookhaven, as well as Lydersen, data were significantly higher than those of Martin, which seems surprising in view of the somewhat greater overall bed voidage of the two former experiments. In this connection, however, it should be pointed out that Martin obtained considerably more data at low Reynolds numbers than either Brookhaven or Lydersen since he used a viscous oil. The size of the Brookhaven equipment precluded the accurate control of small water flows, as well as the precise measurement of low flows and pressures, so that relatively few data with considerable scatter were obtained in the viscous flow region. The Brookhaven data also became independent of Reynolds number above a value of 4,000, while Martin's data, at least up to Reynolds numbers of 6,000, were still decreasing.

The data, uncorrected for wall effects, were in excellent agreement with those of Hendrie (*5*) at identical ball spacings of 1.072 and 1.200 ball diameters; the results at a spacing of 1.154 ball diameters were almost the same as Hendrie's values at a spacing of 1.148, the closest packing to it.

Test beds *F* and *G* were investigated to determine the effect of the container walls, since both have the identical ball spacing and bed voidage, but the former was packed with 1/8- and the latter with 1/2-in. diameter balls. The results obtained for Reynolds numbers in excess of 300 were investigated; low flow data for test bed *F* appeared to be at variance with the rest of the experimental data and could not be explained. The curves for the two beds were not superimposed, since bed *F* which had the smaller overall voidage had a correspondingly slightly higher pressure drop than bed *G*. An attempt was made to bring the two curves closer together by correcting for voidage. Each set of friction factor data was multiplied by the ratio of $(1 - \varepsilon)/\varepsilon^3$ for the bed voidage to $(1 - \varepsilon)/\varepsilon^3$ for the overall voidage. The resulting points for the two curves were in excellent agreement, deviating by <3% from the average.

Test beds *B* and *C*, *C* and *D*, and *E* and *G* were compared by correcting the data of one bed in each pair to the voidage of the other by using the $(1 - \varepsilon)/\varepsilon^3$ correction. The average deviation from the mean in beds *B* and *C*, and *E* and *G*, which differed by ball spacings of 0.012 and 0.016 ball diameters, respectively, was $\approx 3\%$, while that of beds *C* and *D*, whose ball spacings differed by 0.026 diameters, was $\approx 10\%$. This tends to show that the $(1 - \varepsilon)/\varepsilon^3$ correction may be used to correct the pressure drop data

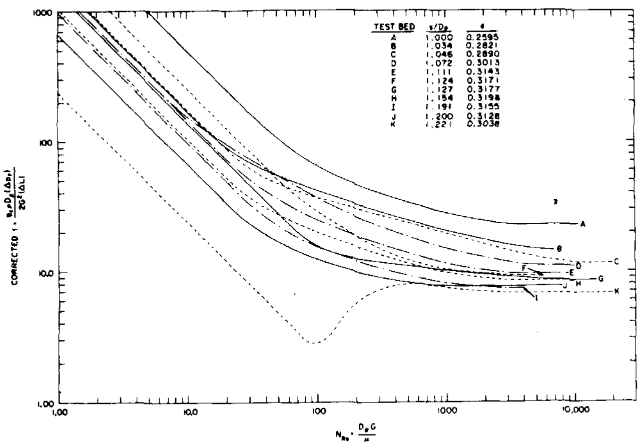

Fig. 5. Pressure drop in ordered packed beds.

for ordered packed beds as long as the difference in ball spacings is not very great. On this basis, then, the wall effect corrections would vary between 1.01 and 1.21 for all the beds investigated.

The friction factor, thus corrected for wall effect, was plotted against modified Reynolds number for all the beds and is shown in Figure 5. The following observations may be made. With the exception of test bed *K*, all the curves showed a relatively smooth transition from viscous to turbulent flow. The friction factor in the viscous flow region for all beds varied as the reciprocal of the Reynolds number, while at high flows the friction factor in all cases became independent of Reynolds number. Although some crossing of curves occurred, the friction factor generally decreased as the ball spacing increased at constant Reynolds numbers in the turbulent region. The crossing of curves generally occurred in the region of 10 to 100 so that a transition effect was certainly present. The curve obtained for test bed *K* (the widest possible spacing for this geometry) is unique, since it exhibited a well defined minimum. Its shape is similar to that obtained for flow in the transition region in round pipes with very low surface roughness.

If one considers the entire geometric system studied, a definite change in the effect of bed voidage on the friction factor may be seen in the regions between test beds *A* and *H* and *H* and *K*. In the first series, the friction factor decreased with increasing bed voidage, while in the second case it decreased with decreasing bed voidage. This explains the increasing deviations between the experimentally measured data for loosely packed beds, and the friction factor data obtained merely by correcting the values of the tightest packing by the appropriate $(1-\varepsilon)/\varepsilon^3$ factors.

It is believed that the curves in Figure 5 can be used generally to determine pressure drop in any ordered bed. The procedure would be to first find the curve most closely fitting the desired conditions, and then correcting it for the new voidage by using the appropriate $(1-\varepsilon)/\varepsilon^3$ factors.

Effect of Voidage

Values for the friction factor, corrected for wall effects, were plotted against bed voidage as a function of modified Reynolds number in Figure 6. As was already apparent in Figure 5, the results showed the presence of two separate systems, one lying between ball spacings of 1.000 and 1.155 ball diameters, and the other between 1.155 and 1.225 ball diameters. The negative-slope portion of the curves in Figure 6 represents the system lying to the left of the minimum packing fraction, and the positive-slope portion that part lying to the right of the minimum in the

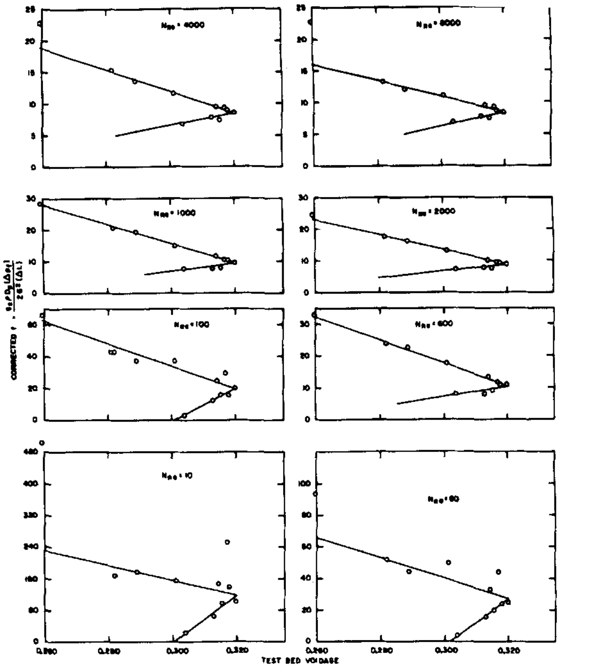

Fig. 6. Variation of friction factor with bed voidage in ordered packed beds.

curve of Figure 1. Both sets of curves intersected at values representing the minimum packing fraction. It is interesting to note here that the slopes of the two sets of intersecting curves varied as the modified Reynolds number but, unfortunately, in a very complex fashion.

Effect of Ball Spacing

The friction factor data, corrected for wall effects, were cross plotted against ball spacing as functions of the modified Reynolds number. Continuous curves were obtained in all cases; however, a straight-line relationship was obtained only beyond a spacing of ≈ 1.040 ball diameters. All curves increased in slope from a ball spacing of 1.040 to one of 1.000. The slopes of the straight-line sections were found to decrease with increasing Reynolds number, becoming independent of Reynolds number above a value of 600. In this high flow region

$$f = 60 - 44\,(s/D_p)$$

The significance of the change in slope of these curves at a ball spacing of ≈ 1.040 ball diameters might lie in the fact that the ratio of free flow bed area to superficial column area increases sharply between 1.040 and 1.000.

Three pairs of packed beds—*D* and *K*, *E* and *J*, and *F* and *I*—were investigated and in each case the bed voidage was approximately the same, but the ball spacing varied significantly. In each pair, the pressure drop for the bed with the greater ball spacing was significantly less than for its partner, even though the overall voidage (including the wall effect) of the latter was significantly greater (Figure 5). The deviation increased markedly as the ball spacing became greater.

Entrance and Exit Effects

The pressure drop was measured as a function of bed height in four beds to determine the extent and magnitude of the entrance and exit effects of liquid flowing through ordered packed beds. No further changes in pressure drop occurred beyond a height of seven ball layers in any bed; none occurred after three ball layers in test bed A. These results are in good agreement with the data obtained by Martin (2), and also showed that neither the ball diameter nor the ball spacing played any significant role.

GENERAL CORRELATION OF THE DATA

An extensive attempt was made to obtain a general correlation of the data which would apply to the entire packed bed system studied. Although expressions could be obtained to relate the mutual effect of bed voidage and ball spacing on pressure drop through the bed, an overall general correlation could not be found which would bring all the curves together, especially in the transition region.

Friction factor curves may sometimes be represented by an equation which adds the relative viscous and inertial effects (*10*). This approach, when used in conjunction with the voidage correction terms, was the only one found to be even moderately successful here, and can be expressed as

$$f[\varepsilon^3/(1-\varepsilon)] = k_1(1-\varepsilon)/N_{Re} + k_2$$

in which k_1 represents the viscous effect and k_2 the inertial. The constants were computed by plotting $f[\varepsilon^3/(1-\varepsilon)]$ against $(1-\varepsilon)/N_{Re}$; straight-line relationships were obtained for each test bed investigated. An average slope of 83.4 (with ±10% average deviation from the mean) and average value for the intercept of 0.42 (with an average deviation from the mean of ±0.02) were obtained. This resulted in the following expression

$$f[\varepsilon^3/(1-\varepsilon)] = 83.4\,(1-\varepsilon)/N_{Re} + 0.42$$

which was then superimposed on all the data in Figure 7. In the viscous and highly turbulent flow regions, the maximum deviation of the data from this line was ≈12%; considerably more scatter was found in the intermediate flow region, approaching ±50%. It is readily apparent that this expression does not completely correlate the data. In addition, it does not correlate the data at all between ball spacings of 1.155 and 1.225 ball diameters.

CONCLUSIONS

The following conclusions may be drawn from this study:

1. An increase in the relative horizontal spacing of balls in the same geometric array, which determines the size of continuous flow channels through the bed, has a very significant effect in decreasing the pressure drop of fluid flowing through the bed. This effect becomes more pronounced as the ball spacing increases, especially beyond the minimum packing fraction.
2. The effect of ball spacing on pressure drop is more important than that of voidage.

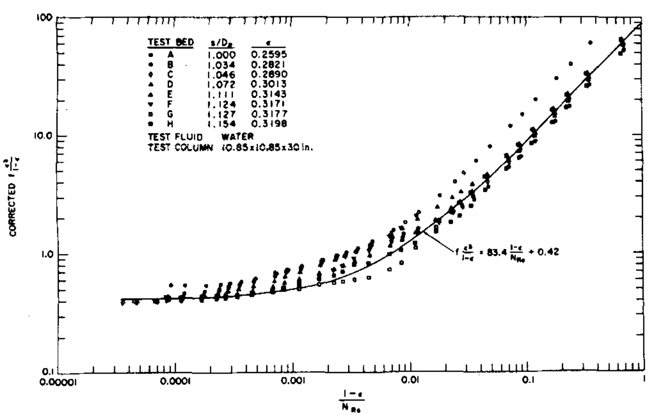

Fig. 7. Correlation of pressure drop with flow in ordered packed beds between ball spacings of 1.000 and 1.155 ball diameters.

3. The data may be corrected for bed voidages, but only over small variations in ball spacings, by the ratios of $(1-\varepsilon)/\varepsilon^3$ at the two voidages.
4. The system studied appears to consist of two separate parts, one region lying between ball spacings of 1.000 and 1.155, and the second between 1.155 and 1.225 ball diameters. The expression

$$f[\varepsilon^3/(1-\varepsilon)] = 83.4\,(1-\varepsilon)/N_{Re} + 0.42$$

correlates the data moderately well, but for the first part only, within ≈12% in the viscous and turbulent-flow regions, and increasing to ±50% in the transition flow region. No correlation was found for the second region.

5. Entrance and exit effects in this geometric system cannot be measured beyond the first seven ball layers in the bed. They were independent of ball diameter and ball spacing.

ACKNOWLEDGMENT

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NOTATION

- b = width of test column, ft.
- D_p = ball diameter, ft.
- e = height of protrusions and valleys on particle surface, ft.
- f = friction factor, $g_c \rho D_p (\Delta p_f)/2G^2(\Delta L)$
- g_c = conversion factor, 32.2 (lb.$_m$)(ft.)/(lb.$_f$)(sec.)^2
- G = superficial mass velocity of water in column, lb./(sec.)(sq. ft.)
- k_1 = constant relating to the viscous effect of flow
- k_2 = constant relating to the inertial effect of flow
- L = bed height, ft.
- Δp_f = pressure drop of water flowing through the packed bed, lb./sq. ft.
- N_{Re} = modified Reynolds number, $D_p G/\mu$
- s = closest center-to-center ball spacing, ft.

Greek Letters

- β = geometry factor of packed bed
- ε = packed bed voidage
- μ = viscosity of water, lb./(sec.)(ft.)
- ρ = density of water, lb./cu. ft.
- φ_s = shape factor of bed packing

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